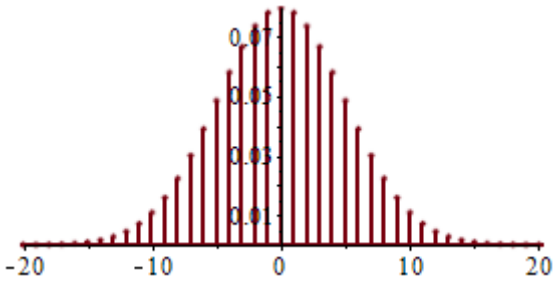
The Random Walk (Galton Board) Theory of Light (Electromagnetic Radiation)

[or the beginnings anyway; it would take another PhD to perfect it.] It is the contention of this research that diffraction patterns of light and other electromagnetic radiation [emr of any frequency], as per Maxwell theory, are patterns of **central limits**, that of the **Central Limit Theorem** (CLT) and the central algorithm of all of (parametric) statistics. This theorem is also known as the **Law of Large Numbers** and says that when random variables are summed (or averaged), the higher order cumulants (skewness and kurtosis for example) become dominated by the lower order cumulants, most notably by the variance. If n random variables are summed, all the cumulants sum linearly (which is why cumulants are the focus rather than central moments for example). Note that sometimes I just refer to cumulants as moments [since they coincide for variance, skewness and kurtosis]. The nth root of the nth order cumulant is then comparable with the square root of the variance, namely the standard deviation σ. Then, providing that the variance is real, all cumulants above second (the variance) are order n, and order $1/n$ relative to the variance; $O(1/n)$. As n becomes large and $1/n$ becomes negligible ($1/n \to 0$), the variance dominates and all higher order variation becomes negligible relative to the variance. This leaves the resulting random variable, as the convolved sum, in the form of a normal bell, the Gaussian bell curve. This is the curve that is seen at the base of a Galton board as all the non-normal peaks and skew average out with repeated sampling, i.e., with a large number of balls falling through the pegs. In the context of the diffusion calculus, the summed random variables (at the pegs) are the **basis components** and the resulting convolved sum (at the base) is the **diffusion variable**. For a Galton board, the basis components are symmetric Bernoulli variables (binomial with parameters $n = 1$ and $p = ½$) and the curve at the base of the board is the transition distribution of the diffusion variable. Each ball has a 50-50 chance of falling to either side of each peg it lands on (hence $p = 1/2$) and the forward transition length is n rows of pegs. At the base, the variable is binomial which becomes indistinguishable from a Gaussian bell, for any largish n, as being that of the Central Limit. This form of the CLT is the **de-Moivre Laplace limit theorem**, that is, the CLT for binomial variables. Since it is symmetric ($p = ½$), the shape of the curve is then that of successive binomial coefficients and over a single row of **Pascals triangle**. **(1)** Pascals triangle is then essentially a **forward diffusing heat equation** and **(2)** what you see at the base of the Galton board of n rows of pegs is then the nth row of Pascal's triangle. For example, with a [well designed] Galton board of $n = 100$ rows of pegs (hence the 100th row of Pascal's triangle), this is what you will see



as plotted in maple for the probability function with the commands

```
> n := 100 : x := seq(j, j = -n/2 .. n/2) : p := seq(2^(-n)·binomial(n, x + n/2), x = -n/2 .. n/2) :
> with(DynamicSystems) : DiscretePlot([x], [p], style = stem);
```

where the forward transition variable is a centered binomial

$$p_n(x) = {}^nC_{x+n/2}\, p^{x+n/2} q^{n/2-x} \qquad x = -\tfrac{1}{2}n, -\tfrac{1}{2}n+1, -\tfrac{1}{2}n+2, \ldots -2, -1, 0, 1, 2, \ldots, \tfrac{1}{2}n-2, \tfrac{1}{2}n-1, \tfrac{1}{2}n$$

where x is the number of steps to the right and $n - x$ is the number of steps to the left. To make centering the distribution easier I have kept the number of rows even [so that $n/2$ is an integer]. The probability function then runs over $n/2$ steps to the left and $n/2$ steps to the right for the walk sample space. These two integer limits (for even n) are the boundaries of the sample space for the n step transition. For $p = ½$, the transition probability function is $p_n(x) = {}^nC_{x+n/2} / 2^n$ and with $n = 100$ is

$$p_{100}(x) = {}^{100}C_{x+50} / 2^{100} \qquad x = -50, -49, \ldots -2, -1, 0, 1, 2, \ldots, 49, 50$$

i.e., the shape essentially being the scaled (by 2^n) binomial coefficients and n^{th} (100^{th}) row of Pascal's triangle. Note that the above histogram is plotted on the restricted range from -20 to +20 for the total sample space running over x = -50 to 50 since the tails for more than ±20 are more than 3 standard deviations ($20 > 3\sqrt{npq} = 3\sqrt{100(0.5)(0.5)} = 15$) from the center line (shifted to mean zero) and are hence essentially negligible. The endpoints of the total range of the binomial sample space ($\pm n/2$) for large n are then essentially infinities ($\pm\infty$) for all intensive purposes (given that all infinities are relative) as per the sample space for the resulting Gauss curve.

So, so far, as interesting this theory is, it is theory which has been around for hundreds of years, since Galton and Gauss, with the further work of Karl Pearson [Galton's student] [and a bit of extra insight from a certain statistician and unpassed PhD]. Here is the point of all this and my contention: diffraction patterns, all importantly including those with side maxima (that is used to argue that light is a wave) are also **Central Limit Theorems**, and Gaussian forward transition distributions, that with harmonic moments. So from theory that I have been developing for a continuous diffusion variable (to complement Maxwell theory and bring it up to date), I know that Gaussian distributions [forward diffusion variables] can have harmonics in the cumulants and I know that this modulates the Gaussian tails, that is, can produce side maxima. Hence, with the assertions made earlier this year in an earlier email (and to the Physics lecturers in the higher year physics laboratories), this is what I expect you can get with a Galton board with ferromagnetic ball bearings/pellets and an oscillating magnetic field over the board (or ping-pong balls down a Galton board with an oscillating vector field [induced by a perpendicular blower] of air over the pegs - less professional but nevertheless I think doable with enough motivation if resources are limited; note that I am suggesting ping-pong balls because the density of ping-pong balls is comparable to the air of the suggested vector field). I think that generating the harmonic with an oscillating magnetic field [which is preferable to ping-pong balls and blowers] is consistent with the electromagnetic basis of light, as per Maxwell, and for example, keeping gyroscopes spinning with magnetic induction; it seems reasonable since it is essentially replicating the quantum effects and physics on the macroscopic scale. It is also consistent with an electromagnetic ether permeating the Universe, that is, an electromagnetic basis [rather than say, a gas phase compound]. This is the basis that all of the best physicists, such as Michealson and Morley, could not accept did not exist. This is evidenced by the microwave background and is essentially the pegs of the random walk, which provides the basis for the wave. A wave carries energy, and the only way it can do this is via a basis. For light, this basis is electromagnetic and the remnant of the big bang, as heavily suggested by NASA instruments, those that detect and study the microwave background. This electromagnetic basis [which is required for my extension of Maxwell] permeates the entire Universe, including and most importantly, the vacuum of space and hence provides the basis for the propagation of emr. I think that NASA should call this the [Universal] *E-B* basis [which is what I was going to call it for the continuous time and continuous sample space theory that builds on Maxwell; this requires a basis function, and this required basis function is electromagnetic (*E-B*, the symbols for electrical and magnetic fields respectively) – gyros spinning in space I think is more evidence of the *E-B* basis].

Doing this physically on a Galton board, and the supporting random walk theory, supports (since it is not yet scientific fact) the contention that quantum mechanics is essentially classical: pure calculus and Maxwell theory extended to an entropic basis function, that is, a Gaussian basis and hence a resulting Gaussian diffusion transition distribution. So here is the theory that I think can be demonstrated on a Galton board:

The step probability is determined at the time the ball hits the *nth* row peg [under the gravitation force, which acts as unit root for propagation down the board]. At that time for a normal walk, it falls to the left (-1) with probability ½ and to the right (+1) with probability ½. Within an oscillating magnetic field, the harmonic at that time (at the time it hits the peg) influences the probability of whether it is a right or left step and biases the odds with factor $0.5\sin(\omega x)/\sqrt{n}$ which is the state of the biasing field at the time the ball hits the peg. From the probability function of the ordinary Galton board

$$p_n(x) = {}^nC_{x+n/2}\, p^{x+n/2} q^{n/2-x}$$

with p = ½

$$p_n(x) = {}^nC_{x+n/2}(1/2)^{x+n/2}(1/2)^{n/2-x} = {}^nC_{x+n/2}/2^n$$

now additionally with the biasing harmonic, the step probabilities become $p = 0.5 - 0.5\sin(\omega x)/\sqrt{n}$ and $q = 0.5 + 0.5\sin(\omega x)/\sqrt{n}$ where ω is the frequency of the biasing field. The probability function becomes

$$p_n(x) = {}^nC_{x+n/2}\left(0.5 - 0.5\sin(\omega x)/\sqrt{n}\right)^{x+n/2}\left(0.5 + 0.5\sin(\omega x)/\sqrt{n}\right)^{n/2-x}$$

which is a form of the suggested transition function [there are several variations on this that I will discuss shortly]. Why I chose this function: Sine is an odd function so the transition distribution (diffraction pattern) is symmetric with the signs of the biasing factors in p and q reversing on either side of the vertical center line (where the number of right steps equals the number of left steps) and hence for the side maxima. This produces a symmetric pattern around the center line, as for diffraction patterns. It is also consistent with what I know about harmonic motions for continuous time and continuous sample space, in particular, the mean and variance. Since sine is bound between -1 and 1, the step probabilities vary between 0 and 1 and the two complementary probabilities add to one

$$p + q = \left(0.5 - 0.5\sin(\omega x)/\sqrt{n}\right) + \left(0.5 + 0.5\sin(\omega x)/\sqrt{n}\right) = 1$$

so it is a valid probability function. The mean for **a one step transition** then depends on the distance from the center line of the board which depends on the phase of the harmonic at that position and step number

$$E(X) = (+1)p + (-1)q = \left(0.5 - 0.5\sin(\omega x)/\sqrt{n}\right) - \left(0.5 + 0.5\sin(\omega x)/\sqrt{n}\right) = \sin(\omega x)/\sqrt{n}$$

which is harmonic. Overall at the forward transition [the base of the Galton board or distance of the photo-detector from the light source and slit], the pattern is centered at the center line. The second raw moment of a step is the same as $p+q$

$$E(X^2) = (+1)^2 p + (-1)^2 q = p + q = \left(0.5 - 0.5\sin(\omega x)/\sqrt{n}\right) + \left(0.5 + 0.5\sin(\omega x)/\sqrt{n}\right) = 1$$

so the variance is

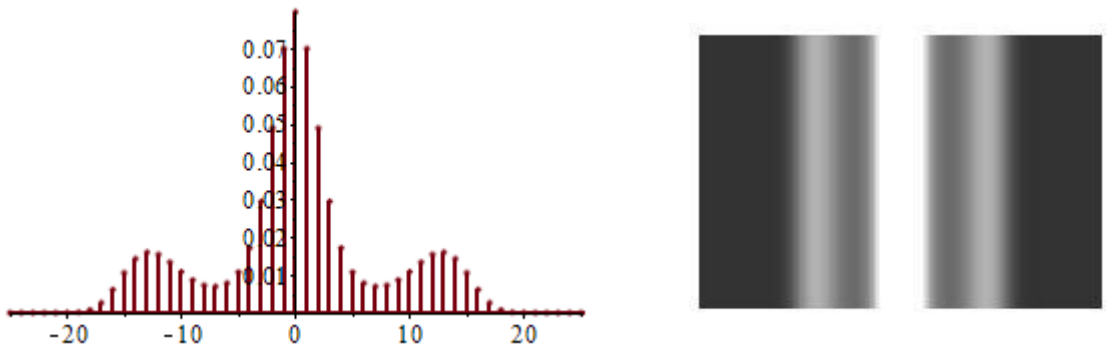
$$Var(X) = E(X^2) - E(X)^2 = 1 - \sin^2(\omega x)/n$$

which is consistent with the variance of a Gaussian harmonic motion [for continuous time and sample space]. These moments (the first two cumulants) then cause reinforcement and cancelation, depending upon the phase of the harmonic, and cause side maxima within the binomial probability bell curve/histogram. I think a good name for this the **Harmonic Central Limit Theorem**.

Plotting the probability function

$$p_n(x) = {}^nC_{x+n/2}\left(0.5 - 0.5\sin(\omega x)/\sqrt{n}\right)^{x+n/2}\left(0.5 + 0.5\sin(\omega x)/\sqrt{n}\right)^{n/2-x}$$

for ***n* =100** rows of pegs consistent with the Galton Board Bell in the first graphic but with the biasing harmonic in the step probabilities



noting the prominent side maxima from the addition of the oscillating probability biasing harmonic. The density plot of this is next to the intensity distribution. This is from balls through pegs which is almost indistinguishable from monochromatic photons through a slit. So I am suggesting that you can do this physically [get this at the base of a Galton board] from the oscillating magnetic field placed behind the board to oscillate the step

probabilities. The oscillating field must not be too strong in relation to the ball bearing mass, otherwise *p* will be 1 or zero [and it will no longer be stochastic], and must not be too weak otherwise the biasing harmonic will not be sufficient to introduce sufficient biasing probability and hence not have the required effect [for example blowers would obviously not work for metal ball bearings]. Therefore the strength of the biasing oscillating magnetic field must be in proportion to the density of, for example, the ferromagnetic ballbearing it is influencing as it falls through the pegs.

Note that for the pattern graphed, the probability biasing harmonic has a total of 5 periods within the walk sample space of $x = -50, ..., 50$. The transverse harmonic is consistent with Maxwell's contention of the transverse *E* and *B* fields within light which cross (×) into its velocity (Pyonting) vector of travel. This then says that light is both a particle (a photon) and a wave (harmonic). The **individual balls** are **definitely discrete particles** and the **many-particle behavior** with the harmonic is **definitely expected from what is considered a wave**. As an example, water is made of H_2O molecules but acts as a wave when, for example, there is many H_2O molecules in motion together as a **many particle system**. This [fluid motion] will also be normal diffusion [and why I boasted while at the Nuclear Science and Technology Winter School at the Australian Light Water Reactor (OPAL) that I could probably solve Navier Stokes].

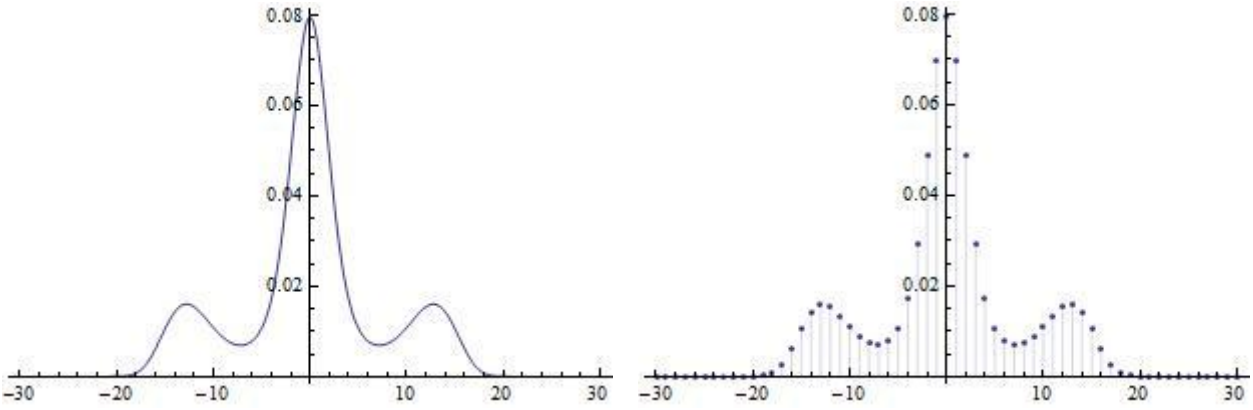
The maple code which plotted this pattern was

```
> n := 100 : x := seq(j - 0.5*n, j = 0 ..n) :
> px := seq(binomial(n, x + 0.5*n) * (1/2 - 1/(2*sqrt(n)) * sin((5*2*Pi)/n * x))^(x + 0.5*n) * (1/2
      + 1/(2*sqrt(n)) * sin((5*2*Pi)/n * x))^(0.5*n - x), x = -0.5*n ..0.5*n) :
> plot([x], [px], -30 ..30, style = point);  (the stem is obtained with the dynamic systems package)
```

To get the intensity plot, the right hand graphic, replicating the actual light diffraction pattern in Encyclopedia Britannica (shown on the next page)

```
> with(plots) : unassign('x','y') : densityplot(binomial(n, x + 0.5*n) * (1/2
      - 1/(2*sqrt(n)) * sin((5*2*Pi)/n * x))^(x + 0.5*n) * (1/2 + 1/(2*sqrt(n)) * sin((5*2*Pi)/n * x))^(0.5*n - x), x = -30 ..30, y = -30
      ..30, grid = [128, 128], style = patchnogrid, brightness = 0.7, contrast = 0.7);
```

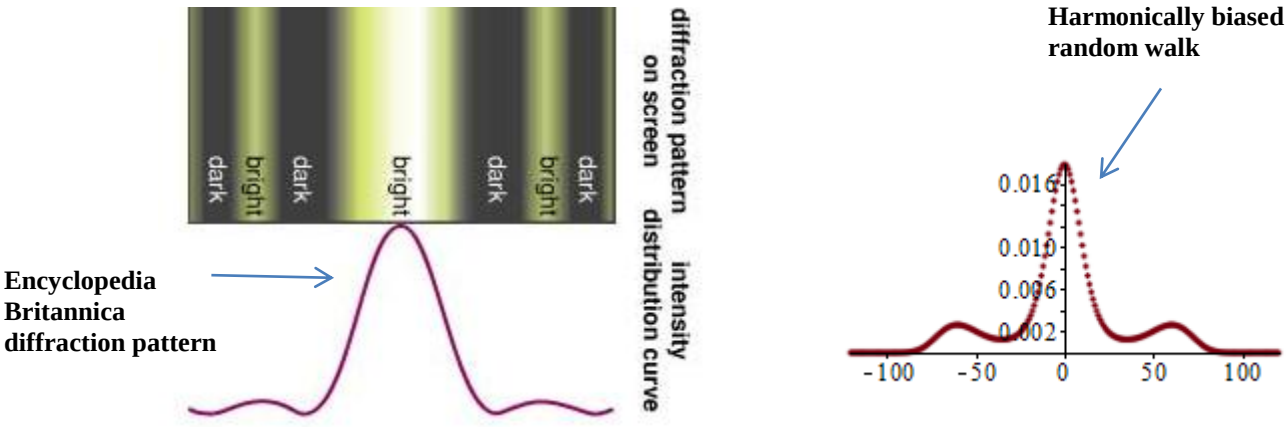
You can also plot this pattern in mathematica




```
Plot[Binomial[100, x + 50] * (1/2 - 0.05 * Sin[5 * 2 * Pi * x/100])^(50 + x) * (1/2 + 0.05 * Sin[5 * 2
    * Pi * x/100])^(50 - x), {x, -30,30}, PlotRange → Full]
```

or any other decent mathematical software package [the other important one being matlab], given that it is just a binomial function with the probability biasing harmonic. Simple and well-designed mathematics is always the best in my opinion; that which makes sense to intuition. And it is even better when it can be **demonstrated physically**, in this case with a Galton Board [although yet to be demonstrated – willing to place bets? I am]. Making and keeping it simple makes bigger and better things possible by making scaling and extensions easier.

Below is a picture of a diffraction pattern in **Encyclopedia Britannica** (2008 Reference Suite) under the topic "***light***" and subheading of **diffraction** (and a ***n* = 2000** random walk pattern next to it)



hence, the **Random Walk Theory of Light** that I have suggested and developed here is very very strong evidence (although not yet definitive proof) for the validity of my assertions, namely that diffraction patterns are **Central Limit Gaussian bell curves with harmonic transition moments** and hence that light is both a particle and a wave – a particle of probability/energy[1] density with a harmonic in its moments. In a liquid or oil drop [how Millikan first measured electron charge and also a current model for muclei], the harmonic transition moments would be the source of reverberating wobble. The maple code to produce the harmonically biased *n* = 2000 random walk probability function that resembles the diffraction pattern in **Encyclopedia Britannica** (which I have taken as a prototypical diffraction pattern) is

```
> n := 2000 : x := seq(j - n/2, j = 0..n) :
> y := seq(binomial(n, x + n/2) * (1/2 - 1/(2*sqrt(n)) * sin((21*2*Pi)/n * x))^(x + n/2) * (1/2
      + 1/(2*sqrt(n)) * sin((21*2*Pi)/n * x))^(n/2 - x), x = -n/2..n/2)
> plot([x], [y], -120..120, style = point);
```

Note that on a Galton board [and any random walk], the sample space of the forward transition is (Pascal's) triangular/isosceles with **(1)** a height of *n* units of peg row (vertical) differences and **(2)** base width of *n* units of peg (horizontal) spacings so that **(3)** the harmonics in the step probabilities whose frequency depends on *n* at the forward transition may arise from either diurnal (time-vertical) or transverse (horizontal-space) harmonics, i.e., longitudinal or transverse. The nature of these harmonics is made clearer with the continuous time theory which is easier and exactly consistent with Maxwell theory [and which I will resolve more fully and probably be typing up next, after the Fourier theory which complements the power series paper/pdf which demonstrated that power series is an example of the diffusion equation (with $n = \infty$ unit roots) as, I am contending, is all of mathematics – the next email send (probably not with an attached paper) will regard enzymes and their transition metal centers for which I will be suggesting that they lower activation energy primarily by means of *d*-block spectroscopic transitions (rather than, say, conformational changes)].

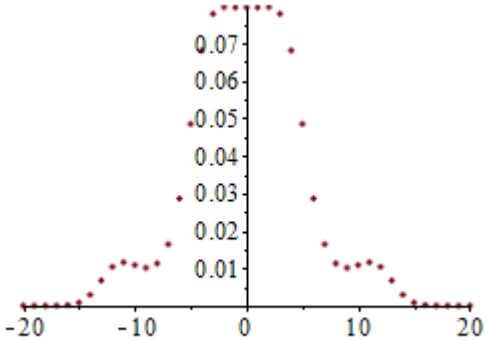
[1] since, I will argue shortly, all matter is Gaussian [by the de Broglie hypothesis] and then, by $\varepsilon = c^2 m$, all is energy is also Gaussian density.

This is complementary and confirmatory of Maxwell's theory of light, which is where I built these ideas from, namely PHYS1121 and PHYS 2111 (which is where Maxwell's theory of light is taught) in the context of my earlier degrees of statistics and actuarial (which is where I learnt mathematical statistics), and my unpassed PhD (where I developed and promoted diffusion calculus of arbitrary [and arbitrarily large] order). When I saw the diffraction patterns, it immediately said to my thinking [just like when I see a premium without a mean μ] that these are Central Limit patterns. Avogadro's number is a very large numbers ($N_A = 6.022 \times 10^{23}$) and so when talking in moles of photons or electrons (or atoms or molecules of ideal gas), there is then plenty of scope for the law of large numbers to come into play, i.e., everything has more than ample chance to become normal via the central limit theory and law of large numbers. Hence, everything being normal bells (as I, and most statisticians, keep going on about). One may ask that if everything is normal bells (including, by hypothesis, normal lifespan and diffraction patterns), then why is it that everything all so different and diverse? The answer is of course because of the form of the cumulants, essentially the flux from the application of the calculus; flux in the mean and flux in the variance, which usually sum together [so it is not as difficult as it may at first seem to extend from *R* to *S*].

It can take quite a lot of adjustments and mucking around [positioning the laser and slit] to get a nice looking diffraction pattern of the prototypical shape. There are so many possibilities, particularly with rotational degrees of freedom and so many variables that can be adjusted. Same with the form of the random walk probability function for fitting the biasing harmonic, and the same with physically designing and placing an oscillating field over a Galton board with ferromagnetic balls; difficult but I think definitely possible. You could probably place current loops or coils through pegs at fixed intervals over each row and run current through them to get the magnetic field. This would look quite sophisticated and interesting. One could also consider inducing resonance in these loops, i.e., with *LC* (Inductor-Capacitor) circuits. As a simple first step(!), it would probably be possible to demonstrate side maxima by simply waving a magnet behind each ball as it walks down the board [which would have to be repeated and repeated to get the large numbers]; the waiving of the magnet introducing the harmonic. This would take time and be repetitive (which is why it would then be automated with a field), but this sort of unsophisticated monotonous work is what people used to do with pendulums to measure *g* [at around the same time as Galton]. It is a good first base, but if someone does have the motivation to try this, they should be sure not to get hypnotized! (like in higher physics on the Kater's pendulum experiment for example). As a bit of history of science and culture, I would say that this is how the idea of hypnotizing people with a pendulum came about [from all those tireless efforts of measuring *g* by counting swings of pendula].

There are many other patterns for a walk with a biasing harmonic, patterns that are not what would be considered prototypical. Some of these non-prototypical patterns are practically identical to some of the patterns I obtained [when I thought I had screwed up] in the higher year physics laboratories in the PHYS2111 Quantum Mechanics laboratory. The relevant experiment was Heisenberg Uncertainty, which utilizes the theory of Bohr's hypothetical microscope and diffraction patterns. The diffraction images (which I don't have with me [and hence can't include] because they are in storage) were obtained from a HeNe laser with a single slit apparatus, a photo-detector and multiplier, and voltmeter. The diffraction patterns were then drawn by hand on graph paper from the voltmeter readings [in the lab book]. The experiment required three diffraction patterns; one I obtained was almost exactly as below, another slightly better, and the third had the nice prototypical shape.

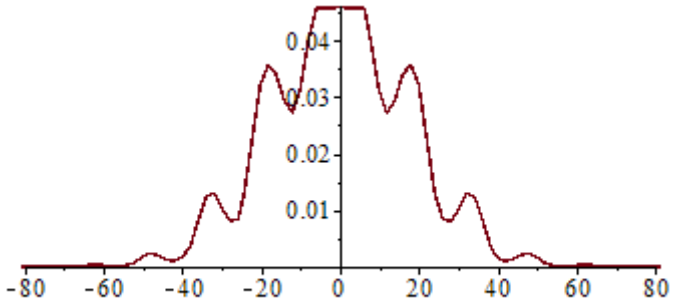
The first diffraction pattern I obtained [the pattern I was least pleased with] was almost identical to the transition probability function as graphed



> $n := 100 : x := seq(j - 0.5n, j = 0..n) :$

```
> y := seq(binomial(n, x + 0.5 n)·(0.50 + 0.02 sin(2 π·0.1 x))^(x + 0.5 n)·(0.50 - 0.02 sin(2 π
      ·0.1 x))^(0.5 n - x), x = -0.5 n..0.5 n):
> plot([x], [y], -20..20, style = point);
```

which shows that all experience, even that which seems unsuccessful or disappointing at the time, may be of value for future resolving [which may be of some consolation to the ex-risk-neutral people[2] – note that the theory here is essentially the binomial model (which is that of Galton) where the (surreptitious[3]) debasing slack variable is replaced by the biasing harmonic]. My data and manually graphed pattern is in storage otherwise I would have scanned it and placed in a side-by-side graphic to demonstrate what a close match it is. I plotted this one in [data] points since this is similarly as that obtained from the voltage data from the Vernier scale adjustments of the position of the photo-detector and photo-multiplier tube [which intercepts the HeNe 630nm laser light]. Just like with actual diffraction patterns, there is a wide variety of shapes that can be obtained by adjusting intensity, position, and slit width and so on. This is similarly so with the walk patterns and parameters of the probability biasing harmonic. As this is just the beginnings of the theory, it is not yet perfected. With the continuous theory, I am much more pleased **but the random walk theory is a necessary complement** because it will be possible to **replicate the results on a Galton board, i.e., physically**. My experience with actual diffraction patterns, as measuring and plotting them, is still at the undergraduate (albeit upper undergraduate) level [in physics], but I have seen electron diffraction images that look similar to this monstrosity [by that I mean an absence of a smooth central peak] of a diffraction pattern [probability function] that a harmonically biased random walk produced

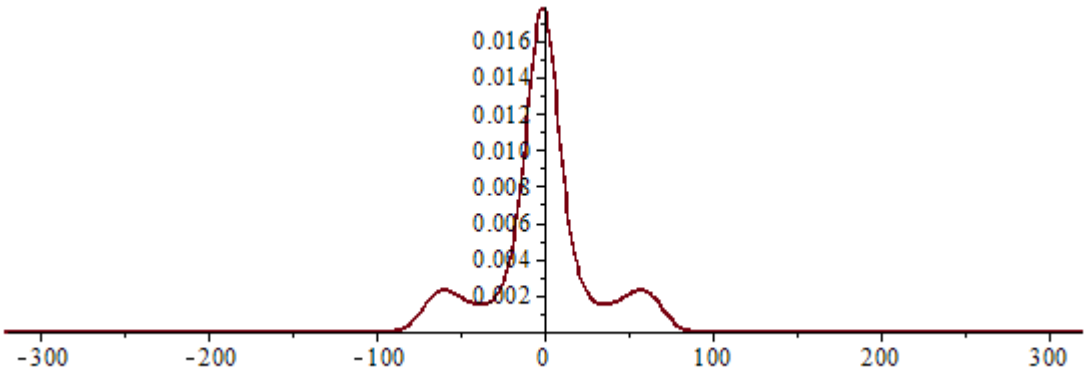


It is my opinion that this is a central limit pattern, more particularly, a **harmonic central limit**. To re-emphasize, this pattern is **definitively a probability distribution** and is **definitively a pattern that is observed in diffraction experiments**. I think that this will be the next big step forward in physics and, I think that all the quantum theory is essentially classical when the extension of the calculus is made from *R* [determinism] to *S* [an entropic basis].

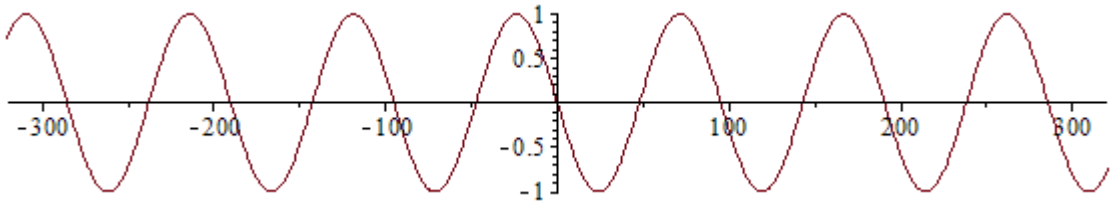
As the Random Walk Theory of Light is in its beginnings, there are some yet unresolved issues not yet perfected, but I expect that it [possible unresolved/unperfected issues] will be [resolved/perfected] in the future. One of the difficulties I was having was getting further side maxima on patterns that I consider prototypical. For example, with the Binomial transition probability function of a hypothetical Galton board diffraction pattern for $n = 2000$, only one side maxima is visible on either side

[2] by this I mean with their calculus knowledge [and knowledge of extending from *R* to *S*].

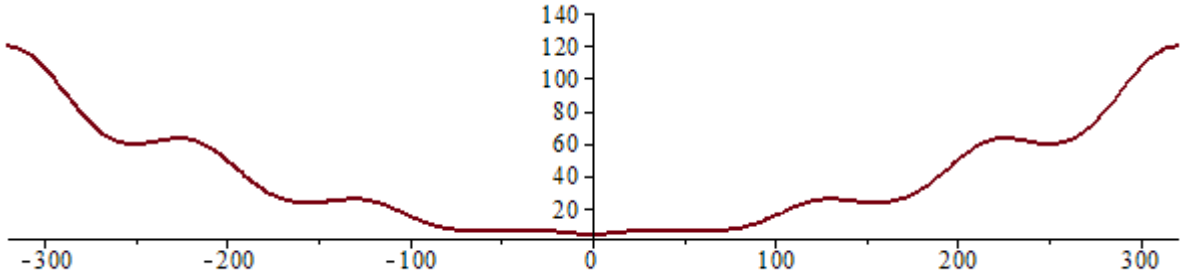
[3] surreptitiously introduced by holding the portfolio flux fixed flat at cash [which debases it and makes the theory communist]. Note that there is no such thing as 'no risk'. Not in this Universe. The best you can ever do is establish a buffer. Anything outside of the buffer, and your position is in trouble. This is the same as all biological systems [the carbonate buffer system of the blood, in my opinion, being the most important example]. Returns are the lifeblood of the [non-communist] financial markets, and a premium for a financial security must always reflect this.



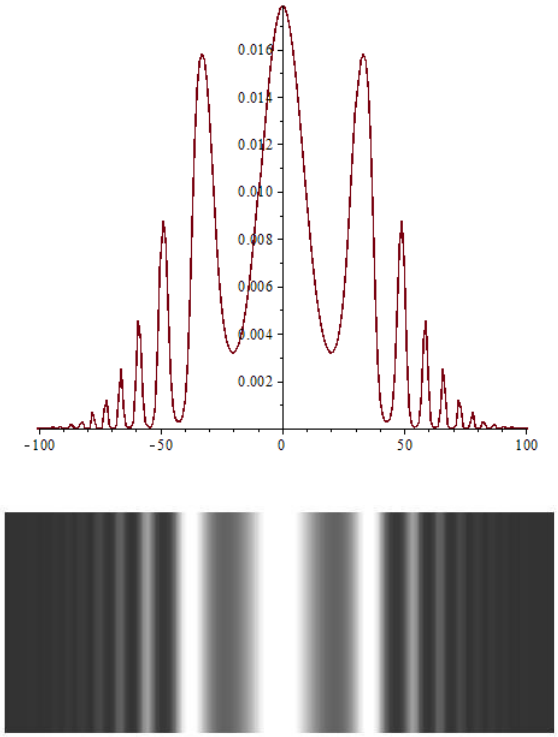
So plotting the probability biasing harmonic in the step probabilities on the same horizontal scale



and taking the logarithm of the walk diffraction pattern shows that the further maxima are there, but just not visible on the ordinary linear vertical scale for the probability;



The turning points for the logarithm of probability show where the side maxima are, including those which are not visible on the ordinary scale [all of them except the first fringe on either side]. To make them more prominent, I considered using the $d\sin\theta = m\lambda$ formula for the positions of **maximum constructive interference** from the light rays diffracted from the edges of the slit, that is, where the **geometric path length difference** was at integer values of the wavelength of the light. So putting an arc-sine function into the probability biasing harmonic makes further side maxima occur earlier [before the decay in the binomial tails dominate]. But this function has a limited range. So I considered a tangent function which is almost the same over the range of the arc-sine. But the tangent function has poles/infinities. So I expanded arc-sine and tangent in a power series to get a better feel for the functions as pertaining to the requirements. The terms for these are all odd. So I tried a polynomial function within the argument of the biasing harmonic, the linear term and a very light third order term. This made further side maxima clearly visible in the walk pattern and density plot. For example,



which is quite a satisfactory diffraction pattern. A nice smooth central maxima with prominent side peaks. The maple code that produced this was

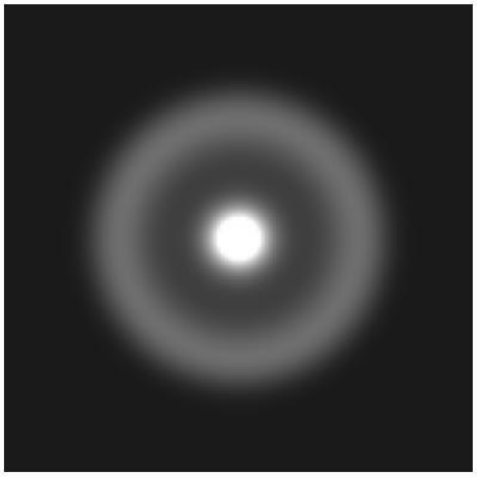
$$n := 2000 : x := seq\left(j - \frac{n}{2}, j = 0..n\right) : y := seq\left(\text{binomial}\left(n, x + \frac{n}{2}\right) \cdot \left(\frac{1}{2} - \frac{1}{2\sqrt{n}} \sin\left(\frac{21 \cdot 2 \cdot \text{Pi}}{n} \cdot \left(x + \frac{x^3}{1000}\right)\right)\right)^{x + \frac{n}{2}} \cdot \left(\frac{1}{2} + \frac{1}{2\sqrt{n}} \sin\left(\frac{21 \cdot 2 \cdot \text{Pi}}{n} \cdot \left(x + \frac{x^3}{1000}\right)\right)\right)^{\frac{n}{2} - x}, x = -\frac{n}{2}..\frac{n}{2}\right) : plot([x], [y], -100..100, style = line);$$

The density plot is graphed with the maple 'plots' package, $with(plots)$ and the graphics command *densityplot*

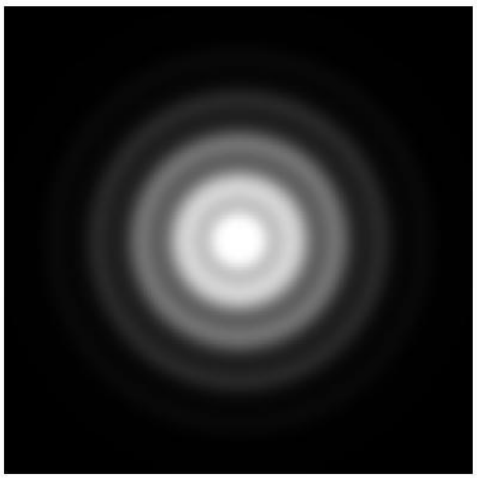
Note that electron diffraction demonstrating waves was first noted by Davisson and Germer, and by G.P. Thompson (1927), and that these patterns were produced even with individual electrons passed through the slit [i.e., discrete particles] by G.I. Taylor. This again implies that electrons, and photons [since it is essentially the same dynamics], are particles with harmonics in their cumulants, i.e., discrete particles that display wave like properties as a many particle system [from their probability distribution properties rather than interactions with other similar particles]. The many particle requirement is hence, not so much for the interactions, but for the law of large numbers to take effect and display the distribution of their [individual] motions and, of relevance for photons, the effects of the probability biasing electromagnetic harmonics [probably interacting with the *E-B* basis].

Also of interest [to me and others] is circular diffraction patterns. Circular diffraction patterns are usually the form of the diffraction pattern obtained for electrons. So as an aside and possible extension, I was considering a Galton board with another dimension, i.e., depth (y), width (x), and height (walk unit root) rather than just width (x) and height (walk unit root). These may already exist but I have never seen one. So for this, I was considering a tetranomial random walk rather than just a binomial random walk. A ball then walks into or out of the page as

well as left or right over the page (or board). These would be layers of offset grids of pegs [offset arrays of holes in perspex layers]. The mathematics of the walk is a composition of binomials. I got the ordinary tetranomial walk going, but the software (maple and mathematica) couldn't seem to handle the biasing harmonics because some of the probability function is not in closed form [it is a combinatorial sum of the arguments, *x* and *y*]. So instead, I just took the *x* variable, considered it as a radius *r* from the origin (or centerline), and put $r^2 = x^2 + y^2$ into the formula for the ordinary binomial walk diffraction pattern. This turned out to be a very nice circular diffraction pattern,



I also did the same with a continuous time and continuous sample space Harmonic Gaussian diffusion variable, which gave the pattern



which is quite pleasing and demonstrative that this theory, the Random Walk Theory of Light [or equivalently, the Galton Board Theory of Electro-magnetic Radiation] has immense potential to put accurate measures on a whole multitude of physical variables, particularly those related to spectroscopy [and interferometry] and those at cyclotrons for example. Australia has one at Monash. Good inexpensive theory can be very valuable to expensive projects and mammoth instruments like that at Monash [and those in the States and Europe]. It is good and simple theory that made them possible in the first place. I find it all very interesting anyway, given that I am a statistician and hence like random walks and the Central Limit Theorem, and a recent full appreciation of light. This theory, with the further aspect of harmonics influencing the walk probabilities, gives such a theory a whole new dimension, essentially that which was hencepreviously out of scope of the theory of random walks – this extension is Brownian motion [random walks] meets Fourier's Theorem, or put in another way, probability theory meets [higher order] calculus. Whenever something is labeled 'Fourier', it is based on an underlying diffusion equation; the most relevant examples here being Fourier spectroscopy (FFT IR and NMR for example) and Fourier Optics.

From the email I sent earlier in the year titled "***Regarding Galton Boards and Diffraction Patterns****"*, the **Random Walk Theory of Light** [and the complementary continuous sample space theory] essentially implies that

- Light consists of particles (photons) that **act as a wave** as a **many particle system** (like H_20 molecules and water waves) through an [Universal] electromagnetic basis;
- the motions of the individual photons or electrons (as single particle considerations) are Gaussian (bell curve) distributed, i.e., Gaussian sphericals [bells in 3D; a bell on *x*, a bell on *y*, and a bell on *z*, which then makes a Gauss spherical in (*x*,*y*,*z*)];
- The Gaussian transition distribution of photons [and electrons] have harmonic cumulants, i.e., harmonic mean and variance that is consistent with the theory of Maxwell [note that in the context of a Gaussian spherical, the problem of zeroes at the nodes is resolved];
- Settles the ambiguity of whether light is a particle, a plane wave, or a spherical wave; it is all three, notably, a Gaussian spherical that travels with unit energy (and hence unit energy density $\varepsilon = hf$) with the two transverse *E-B* dimensions postulated by Maxwell carrying the harmonics [Note that a photon is not an expanding sphere and, consequently, I think that Lorentz boost including time dilation and length contraction are slack variables (just like in finance[4])];
- Diffraction and interference patterns are essentially visual displays of the **Harmonic Central Limit Theorem** [named by yours truly], the CLT and de-Moivre Laplace Central Limit extended to harmonic variables; this is the central algorithm of all of statistics (and, I think, the Universe); that is, photons are Poisson summed Gaussian motions (that you see in search lights and in Poisson diffusion simulations);
- Provides a "mathematical why" behind the results of Davison and Germer, and G.P. Thomson (J.J. Thomson's son) for diffraction patterns for electrons, and would imply that de-Broglie waves are also harmonic transition cumulants so that **all matter** is essentially **Gaussian density** (the dynamics for which being pure calculus, calculus extended from *R* to *S*);
- Given matter-energy equivalence $\varepsilon = mc^2$ [which could be inferred directly from Maxwell theory that **permittivity** and **permeability** are inversely proportional where the constant of proportionality is the square of the speed of light, c^2], that if all matter is Gaussian via the de-Broglie hypothesis in the context of the diffusion equation, then **all energy** is also **Gaussian density**. That makes everything.
- To reiterate, (1) **all energy is Gaussian** (2) **all matter is Gaussian** hence (3) **everything is Gaussian** and takes the form of either **diffusion** [as per **the diffusion equation**] or **heat** [as per **the heat equation** of Fourier, and Pascal's triangle in the context of Random Walks]. This is supported by the Standard Cosmological Model with the evidence of the red shift of an expanding Universe [**diffusion**] and cosmic microwave background [**heat**].

This is the calculus extended from *R* to *S.*

Conclusion

I think that this theory, **the Random Walk Theory of Light**, is an initial step [with what I have in mind] to **extensive modeling of light with diffusion equations**; that is, with the theory of Maxwell in conjunction with Gaussian transition distributions. This is the diffusion equation of arbitrary order, or more particularly, of arbitrarily large order. I think the Random Walk theory is also consistent with the discrete particle-like nature (discrete ball bearings), and diffuse and stochastic like nature (from the walks on the Galton board), of the quanta; light, as individual photons acting as a wave as a many-particle system [through an electromagnetic basis], for example and of relevance here, but anything really, anything that has a de-Broglie wave [which, again, is everything].

Hence, reiterating, I think that these mathematical diffraction images show almost definitively that the calculus extended from *R* [determinism] to *S* [an entropic basis] is that which will explain everything, most notably, the laws of the physical universe [and everything in it].

[4] which stood in the way of upping the [differential] order [and hence sophistication].

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